A MATHEMATICAL-PHYSICAL MODEL OF THE GENESIS OF THE ELECTROCARDIOGRAM

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ABSTRACT A fundamental problem of cardiac electrophysiology is that of relating quantitatively the electrical activity within the heart to the complete time-varying potential distribution at the body surface. A new numerical method is described for the calculation of the surface potential on an irregularly shaped closed external surface due to an arbitrary source distribution in a medium containing regions of different conductivity, subject to the appropriate boundary conditions. The method is intended to provide an exact theoretical analysis of the experimental data acquired by A. M. Scher and others who have been mapping the pathways of ventricular depolarization in dogs and other animals. In anticipation of the above research program, a number of exploratory computations are reported. For example, the surface potential distribution has been calculated for a cylinder of human torso cross-section with a hemispherical dipole layer current source in approximate heart position and orientation and containing "lungs" of conductivity different from that of the surrounding medium. Under certain conditions, when lung-like inhomogeneities are introduced, a simple dipole source can generate a potential distribution having the multiple maxima and minima characteristic of higher multipole sources.

INTRODUCTION

This paper is concerned with a fundamental problem of cardiac electrophysiology, that of relating in a quantitative way the electrical activity within the heart to the complete time-varying poential distribution at the body surface. A solution to this problem is prerequisite to full understanding and interpretation of the electrocardiogram, not only to extend the clinical value of the standard ECG, but also to place well defined limits on the information concerning the activity within the heart that one can ever hope to extract from a surface record. More specifically, we should like to be able to predict the ECG recorded by any set of standard or non-standard leads given a complete description of the time-varying current sources and sinks within the cardiac volume.

Experimental investigation of the detailed cardiac electrical activity has been undertaken by Scher and Young (1) at the University of Washingtion in Seattle, Sodi-Pallares *et al.* (2) at the National Heart Institute in Mexico City, and Durrer and van der Tweel (3) at the University of Amsterdam. Each of the latter groups has devised its own system whereby electrodes are plunged into living animal hearts in order to determine experimentally the pathways of electrical excitation within the ventricular walls. The QRS complex of the surface electrocardiogram has then been qualitatively related to the measured heart activity.

Attempts at a theoretical analysis and quantitative understanding of the above experimental data have thus far met with indifferent success, for the physical problem to be solved is an extremely difficult one. In brief, one must calculate for an arbitrary source distribution in a medium containing regions of different conductivity the complete potential distribution on an irregularly (torso) shaped external surface, subject to the boundary condition that the generated currents be tangential to the external boundary and continuous through all internal interfaces between media of different conductivity (*i.e.* the normal component of the electric field vanishes at the external boundary, and the electric field is refracted at the interfaces).

Among the more realistic configurations for which the surface potential distributions have been computed analytically are the following. Okada (4) has considered the case of a homogeneous cylinder of finite length containing an eccentric dipole generator. Geselowitz (5) treats a number of spherically symmetric configurations, including the case of concentric spheres of different conductivity where the generator is an eccentric dipole or a portion of a concentric spherical dipole layer. The surface potential produced by an eccentric dipole in a homogeneous ellipsoid of revolution has been computed by Martinek and Yeh (6) and independently by Chu (7). In all of the aforementioned work, the solution of the Poisson equation satisfying the boundary condition is constructed as an infinite series expansion—in Bessel functions for the cylinder, and in Legendre functions for the sphere and ellipsoid.

In pursuit of a more useful model of the heart in a torso, a number of workers have constructed "tank models" of the torso in which physical dipoles are immersed in the conducting medium and potentials are measured on the surface. We mention in particular the investigations of Burger and von Milaan (8) and of Frank (9). A somewhat different approach has been followed by Taccardi (10), who has immersed living turtle hearts in cylindrical tanks of conducting fluid and measured the potential distribution produced by a living heart generator in an idealized environment.

POISSON'S EQUATION FOR INHOMOGENEOUS VOLUME CONDUCTORS

Our approach to the problem has been to construct an exact numerical solution to the Poisson equation subject to the boundary conditions satisfying the requirements of the physical system. The method of solution is believed to be novel (11) and may be briefly described as follows. Consider an arbitrary ensemble of generators in a closed, three-dimensional region of electrical conductivity σ. The region is assumed to be embedded in an infinite dielectric. Contained within the primary region are an arbitrary number of regions of conductivity σ_i. There is no restriction upon the configuration of the secondary regions—some may be contained within others, and they may be in contact with one another or with the surface of the primary region (Fig. 1). In the case of our application to cardiac electrophysiology, the primary region is the animal torso embedded in air (the infinite dielectric). The most important secondary regions to consider will, in all probability, be the lungs, of conductivity equal to approximately one-half that of the primary region, and the volume of the heart filled with high conductivity blood (12). The generators are those regions of the myocardium undergoing depolarization at a given instant.

Our method of solution for this boundary value problem in electromagnetic theory follows closely the course of events in the real physical situation. When the generators are turned on, the instantaneous field produced is that of the generators in an infinite conducting medium, with boundary conditions unsatisfied. Electric charge begins to flow immediately along the potential gradient, collecting at all boundary surfaces and interfaces. A steady-state condition is reached when

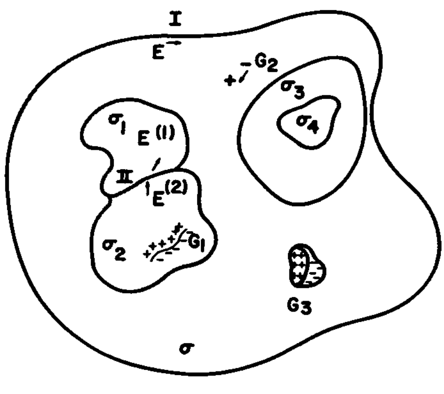

FIGURE 1 A schematic diagram of the configuration treated in this paper. The primary region has conductivity σ while the embedded regions have conductivities σ_1, σ_2, . . . *etc.* The G_1, G_2, . . . are given generators. They may be point sources or sinks, dipoles, dipole layers, higher multipoles, or volume source distributions. The electric field E is tangential to the external surface; *i.e.*, E_{normal} (I) $= 0$. The electric field is refracted at all internal interfaces as at II such that $E^{(2)}_{normal}$ (II)/$E^{(1)}_{normal}$ (II) $= \sigma_1/\sigma_2$. The tangential component of E is continous at the interface.

the fields produced by the boundary surface and interface charge distributions combine with the generator fields to satisfy the boundary conditions everywhere. For a conducting body with the physical characteristics of a human torso, the time constant for the decay of transients is at least an order of magnitude smaller than 10^{-4} seconds. Since the QRS component of the electrocardiogram contains no measurable energy beyond, let us say, 10 kc, the boundary conditions are satisfied in effect instantaneously in the physiological case. This point is an important one, for we intend to approximate the continuously time-varying field configuration with a sequence of steady-state "snapshots" calculated at closely spaced successive time intervals.

The first step of the solution procedure, then, is a calculation of the electric field produced by the ensemble of generators at all surfaces and interfaces where boundary conditions must be satisfied. In the next step (the first iteration), electric charge, as determined by the divergence theorem, is distributed on all boundary surfaces and interfaces such that the boundary conditions are satisfied at each boundary area element. In particular, if E_n is the normal component of the infinite-medium generator field at a given area element, then for an external surface element, a surface charge density of $\omega = E_n/2\pi$ will completely cancel the normal component of the generator field, while a surface charge density of

$$\omega = \frac{E_n(\sigma_1 - \sigma_2)}{2\pi(\sigma_1 + \sigma_2)}$$

will make the ratio of the normal components of the electric field in the two media of conductivity σ_1 and σ_2 surrounding an interface equal to the inverse of the conductivity ratio for the two media.

The charge that has been placed upon a given area element i, however, produces a perturbing field at every other area element $j \neq i$. In the following step, then, the perturbing fields at each area element i produced by the charges at every other area element $j \neq i$ are added up, and additional charge is placed upon element i to compensate for the perturbing fields, so that once again, the boundary condition at i is satisfied (the second iteration). Each time a new layer of compensating charge is distributed on the boundary surfaces and interfaces, a new (and hopefully, smaller) set of perturbing fields is produced, which must in turn be corrected by new charges, for as many iterations as necessary to produce convergence. When convergence has been attained (*i.e.* the perturbing fields have become sufficiently small at every area element), the potential at any point in the system may be calculated by scalar addition of the potentials due to the generators and all of the compensating charges that have been distributed throughout the system. When calculating the potential at a point on a boundary surface, the potential due to the charge on the area element where the potential is being computed must be separately treated to avoid a non-physical divergence at that point.

We remark in passing that the problem of simultaneously satisfying the boundary conditions on a finite number of area elements, which is described above in terms of a sequence of easily grasped physical processes, may be conveniently expressed as a set of m simultaneous linear equations, where m is the total number of area elements on all surfaces and interfaces. In this representation, the problem is seen to be that of inverting a square matrix of rank m. As we have indicated in the foregoing discussion, our method makes use of an iterative procedure to invert the matrix.

THE COMPUTATION

A program has been written for the IBM 7090 high speed electronic digital computer to perform the calculation described above. The computation was checked by comparing the results of our program with the solution for a number of analytically calculable cases. One such comparison is illustrated in Fig. 2 for the case of a dipole source on the axis of an ellipsoid of revolution, with the dipole below the centroid of the ellipsoid at a distance equal to one fifth the minor axis. The analytical calculation is taken from Chu (7), after a slight error in Chu's computation of the numerical values had been corrected. It will be seen in the figure that the

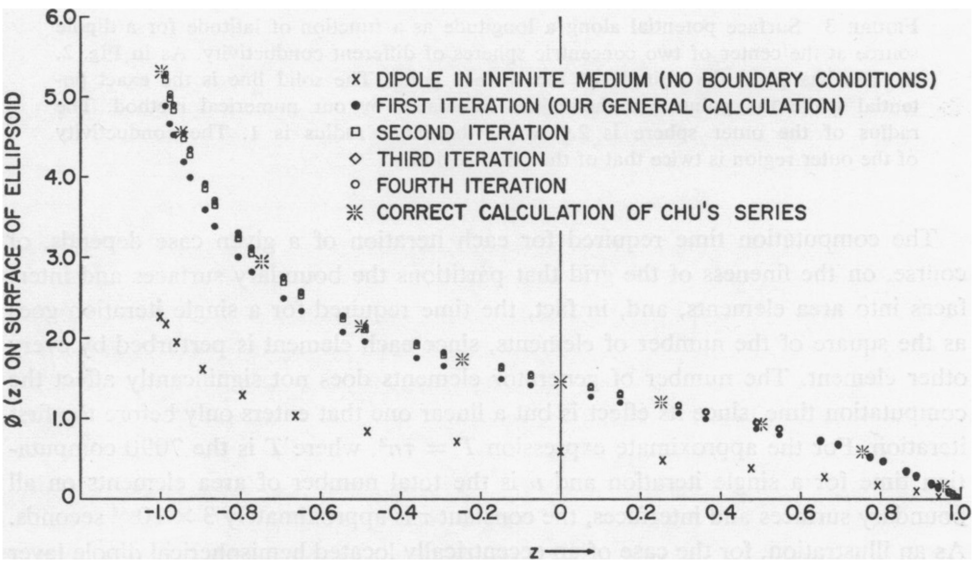

FIGURE 2 Surface potential along a longitude as a function of latitude for a noncentric unit dipole source on the axis of an ellipsoid of revolution. The abscissa gives the surface point in terms of its value on the Z axis, *i.e.*, the altitude of the point, which is the cosine of the latitude. The potential is given relative to the value at the north pole. Our numerical calculation is compared with results of summing Chu's series.

second iteration brings our value to within about 1 per cent of the true value, while the third and fourth iterations, on the scale of our graph, are concurrent and indistinguishable from the analytically calculated values.

A second case that has been calculated analytically is that of a dipole at the center of two concentric spheres, with the conductivity of the central region different from that of the region between the surfaces of the two spheres (Fig. 3). Here again, three iterations are quite adequate for satisfactory convergence.

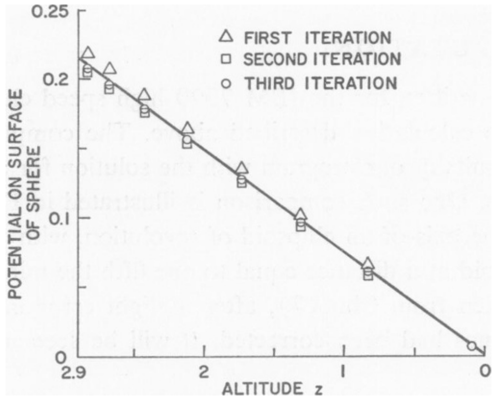

FIGURE 3 Surface potential along a longitude as a function of latitude for a dipole source at the center of two concentric spheres of different conductivity. As in Fig. 2, the abscissa gives the altitude of the surface point. The solid line is the exact potential while the points are the values calculated by our numerical method. The radius of the outer sphere is 2.9 units; the inner radius is 1. The conductivity of the outer region is twice that of the inner region.

The computation time required for each iteration of a given case depends, of course, on the fineness of the grid that partitions the boundary surfaces and interfaces into area elements, and, in fact, the time required for a single iteration goes as the square of the number of elements, since each element is perturbed by every other element. The number of generator elements does not significantly affect the computation time, since its effect is but a linear one that enters only before the first iteration. For the approximate expression $T = \tau n^2$, where T is the 7090 computation time for a single iteration and n is the total number of area elements on all boundary surfaces and interfaces, the constant τ is approximately 3×10^{-4} seconds. As an illustration, for the case of an eccentrically located hemispherical dipole layer current generator in a cylinder of elliptical cross-section (Fig. 5), using a rather fine mesh of 612 area elements on the surface of the cylinder and 56 area elements on the (much smaller) hemispherical current generator, each iteration required about 2 minutes of 7090 computation. We wish to emphasize the point that our solution procedure is, within limits, indifferent to the shape of the surfaces, interfaces, and current generators, as well as to their location and to their method of definition. The computation of a given iteration is no more complicated or time-consuming for an irregular, numerically defined thorax-shaped surface than for a smooth, analytically defined one.

Questions of convergence are difficult to answer in general for a procedure that is intended to encompass so wide a variety of physical configurations. Particularly concerning rate of convergence, the formal theory of matrix iterative analysis offers few strong results that are generally applicable to our problem. Since, in normal use, no exactly calculated result will be available for comparison purposes, the following arbitrary criteria have been established to determine when satisfactory convergence has been achieved:

1. The difference in potential between the maximum and minimum points changes by less than an acceptably small amount from its value at the last iteration.
2. The normal component of the electric field at each surface element has reached an acceptably small percentage of the maximum normal field produced by the uncompensated generator (*i.e.* the boundary condition at the external surface is acceptably satisfied).
3. The surface potential map has become essentially stable (*i.e.* the equipotential lines and maximum and minimum points have not changed their position noticeably since the last iteration).

Just what constitutes "acceptably small" in the specifications above depends upon the use one has in mind for the computation. Where quantitative predictions are to be made for comparison with experimental results, the precision of the experimental data is the determining factor. For purely theoretical work, one would probably want to iterate to the point where one could feel confident that the solution is within 1 per cent of the exact result. For the sample calculations presented below, we were satisfied when the potential map became stable. This generally occurred while the magnitudes of the potential differences were still changing very slowly, although in all cases, we estimate that we are within a few per cent of the final value.

Concerning the rate of convergence, we can only draw some tentative conclusions based on our limited experience with the program to date. For relatively simple configurations (*i.e.* homogeneous conductors of torso shape, or the case where only lung-like inhomogeneities are introduced with relatively small conductivity ratios of the order 2:1), two to four iterations give satisfactory convergence. On the other hand, a configuration containing a high conductivity blood mass (eight times as conductive as the torso medium) where the generator was a hemispherical dipole layer partially enclosing the blood mass required forty iterations before convergence was satisfactory.[1]

[1] *Note Added in Proof.* We have learned, through experience in running a number of configurations since this paper was written, that cases requiring more than four or five iterations

In order to deal with those cases requiring a great number of iterations, several procedures are currently being developed to increase the convergence rate as well as to decrease the computing time required for a single iteration. For example, once the transient effects of the iteration procedure have died out and a definite trend in the charge increments applied to each area element has become discernable, one may introduce a procedure that extrapolates past several intermediate iterations before doing a corrective iteration. In this way, computer time required for problems calling for many iterations may be reduced by a factor of about three.

A final point is the following. When the problem calls for the calculation of a sequence of potential maps at successive time intervals, the generator will have changed only slightly at time t_i from its configuration at time t_{i-1}. The surface potential distribution, too, will in general have changed only slightly during that time, so that the surface charge distribution at time t_{i-1} may be used as a trial function with which to begin the next iteration, materially decreasing the number of iterations necessary for convergence.

A MODEL OF THE GENESIS OF THE ELECTROCARDIOGRAM

The main burden of our research program is to provide an exact theoretical analysis of the experimental data acquired by those workers who have been plotting the pathways of ventricular depolarization in dogs and other animals by the technique of inserting multiple electrodes into the cardiac muscle of living animals. In particular, an active collaboration has been established with Professor A. M. Scher of the University of Washington, who will soon have a 50-channel apparatus available for the detailed exploration of the time-varying electrical activity within the volume of the heart muscle. Scher's experiments will provide us with a precise map of the location of the "depolarization front" for each instant of the QRS complex. It is our intention to use this data to compute the complete surface electrocardiogram for the animal under study (taking into consideration the inhomogeneous resistivity of the thorax medium as far as necessary), and to compare the computed result with that measured on the same animal at the time of the experiment.

Once the validity of our model has been established, it may be used to investigate the effect on the surface electrocardiogram of various simulated cardiac infarctions by making the appropriate modifications in the ventricular depolarization map. In an obvious way, the model will provide a reliable determination of the maxi-

for convergence are usually characterized by a small number of troublesome area elements. These elements occur in regions where the surface charge density required to cancel the generator field is considerably larger than average. By subdividing such area elements to reduce the total charge per element to a value commensurate with the average charge on the other elements, the total number of iterations required for convergence is reduced drastically.

mum information available in the full surface electrocardiogram, as well as an indication of the optimum placement and number of leads necessary to extract that information.

It is expected that our program may be useful in other approaches to the problems of cardiac electrophysiology; for example, the study of lead systems in vectorcardiography. In other areas of electrophysiology, too, our program might find some application. The field of electroencephalography is one that comes immediately to mind as a possibility.

PRELIMINARY RESULTS

In anticipation of the research program described above, a number of exploratory computations have been performed in order that we might gain some insight into the physiological problem as well as experience in the use of our computer program. Calculations were performed for various configurations of the following. The exterior boundary, simulating the torso, was taken to be a cylindrical form of either elliptical- or torso-shaped cross-section. The generator, placed at the approximate location of the heart in the torso, was taken either as a unit dipole in one of two possible orientations or else as a hemispherical dipole layer of unit intensity to simulate a possible isochronous depolarization surface, or "front," in the heart. In some of the cases considered, two closed internal regions of half the conductivity of the surrounding medium (12) were introduced to simulate the lungs. These, too, were cylindrical forms, half as high as the "torso" and having the cross-sections of one left and one right lung. One calculation was performed with a secondary region of eight times the primary conductivity in the form of a hemisphere concentric with the hemispherical current generator to investigate the short-circuiting effect of the high conductivity blood in the heart cavities (12). The "blood-filled" hemisphere, three quarters the radius of the generator, was oriented in the same direction as the outer hemisphere. Figs. 4 and 5 represent two typical configurations from among those described above.

We shall not dwell upon the details of our numerical calculation, but it should be recorded here that the curved surface of the external cylinder was divided into 576 area elements (sixteen bands parallel to the base with each band subdivided into thirty-six triangular elements), while the caps of the cylinder were each divided into eighteen triangular elements. A small section of the mesh is drawn in explicitly in the lower left hand corner of Fig. 6*b*. The x in each triangle represents the centroid location, the point where the field is calculated, and the compensating charge applied. The more or less irregular surfaces of the internal regions and dipole layer generator were triangulated into area elements consistent in size with those on the external boundary. Where the precision afforded by a uniform grid of the fineness adopted for the external surface is felt to be inadequate for a particular computation, we remark that it is entirely feasible within our program to use a grid

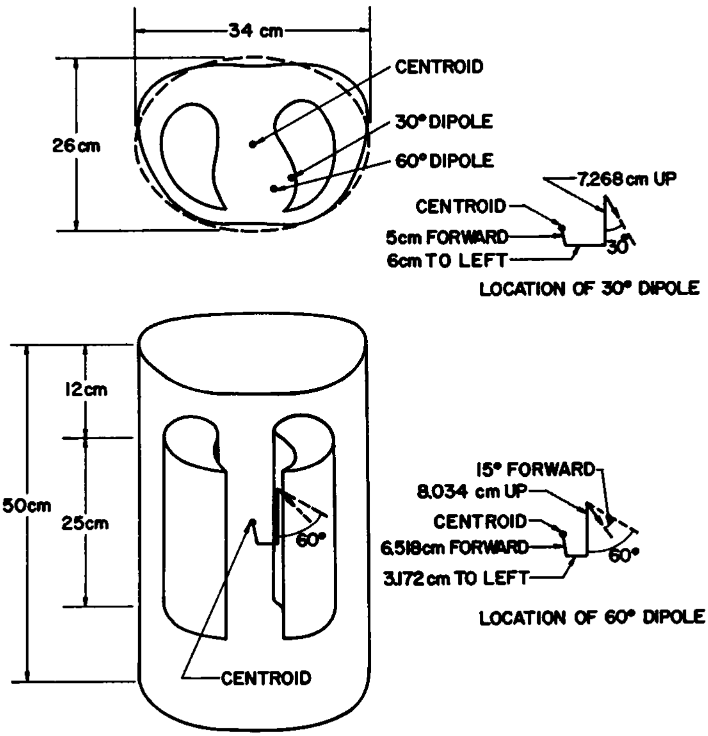

FIGURE 4 The configuration used in the calculations of Figs. 6*c* and 8*d*. The outer surface is a 50 cm high cylinder of torso cross-section. The dashed curve in the top view is the cross-section for the elliptical cylinder. The lungs are also cylinders, 25 cm high, with the indicated cross-sections and with half the conductivity of the surrounding medium. Two different unit dipole generators were used. The "60° dipole" is located 6.518 cm forward, 3.172 cm to the left, and 8.034 cm up from the centroid of the torso. From an orientation pointing vertically downward, the dipole is rotated 60° to the left in the frontal plane and then 15° forward. The "30° dipole" is 5 cm forward, 6 cm to the left, and 7.268 cm up from the torso center. The dipole is rotated 30° to the left in the frontal plane from the downward vertical direction.

of varying fineness on any boundary entering into the calculation. Thus, by subdividing the appropriate part of the surface into smaller area elements, it is possible to secure greater detail on the map where the surface gradient of the potential is high. This procedure was followed to compute the map in Fig. 7*c*. Finally, we would suggest that the details of the potential distribution close to the upper and lower caps of the external cylinder be taken somewhat less seriously than the body of the map away from the upper and lower margins, not only because these first approximation torsos depart excessively from physiological reality at the caps, but also because of the mathematical discontinuity in the normal to the surface

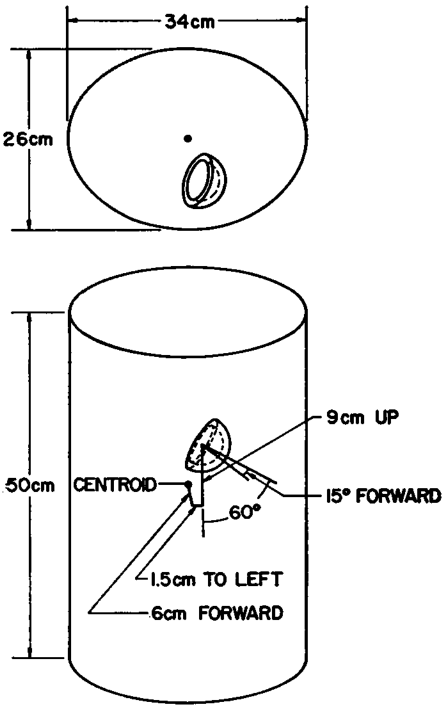

FIGURE 5 The configuration used in the calculation of Fig. 7*d*. The outer surface is a 50 cm high cylinder of elliptical cross-section. The generator is a hemispherical dipole layer of unit strength per unit area and with a radius of 4 cm. The center of the hemisphere is located 6 cm forward, 1.5 cm to the left, and 9 cm up from the center of the cylinder. The axis of the hemisphere is rotated 60° to the left in the frontal plane and 15° forward. The 60° dipole of Fig. 4 is pointed in the same direction as the axis of the hemisphere and is located on the axis half way between the center and the apex. This same hemispherical dipole layer was used for the generator in the calculations of Figs. 7 *a-c*. For Fig. 7*d*, the higher conductivity blood mass was introduced. This was a hemisphere concentric to the dipole layer and with a radius of 3 cm. The purpose was to simulate the high conductivity blood within the heart cavity.

at the rims. Both of these difficulties will be satisfactorily resolved in later calculations when true anatomical boundary surfaces are introduced into the calculation.

Fig. 6 then, displays a series of three surface potential distribution maps resulting from the same current source embedded in three different "anatomical" configurations. The generator in this case was the 60° dipole located as indicated in Fig. 4. The potential distribution produced by the latter dipole was calculated first on the surface of an elliptical cylinder, then on a torso-shaped cylinder, and finally on the torso-shaped cylinder with lungs introduced as a secondary region. In Fig. 7 the potential distributions are calculated for the same three anatomical configura-

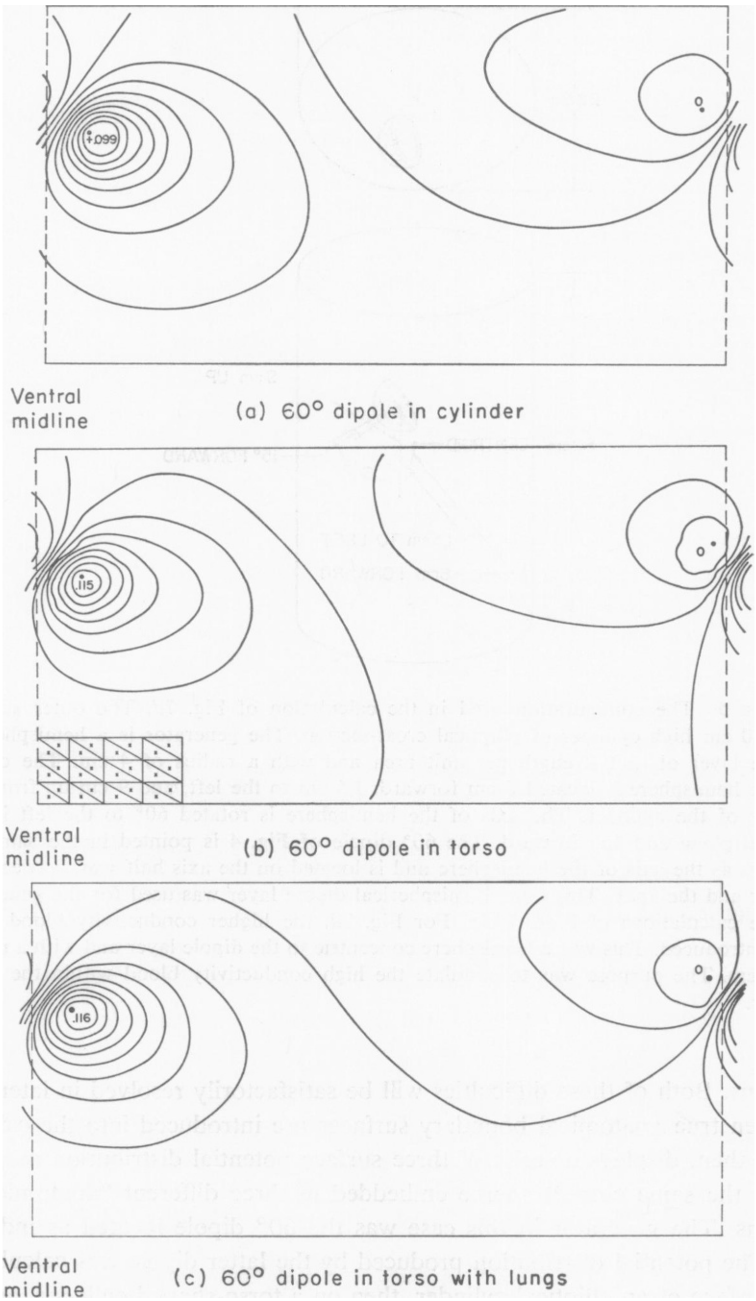

FIGURE 6 Calculated potential maps using the 60° dipole generator described in Fig. 4. The minimum point on each map was set equal to zero potential.

tions but with the dipole generator replaced by a hemispherical dipole layer, centered at the same location, and oriented so that the dipole component of the hemispherical generator distribution coincides with a single 60° dipole. Thus, the 60° dipole could be considered to be the "equivalent dipole" for the hemispherical dipole layer.

Fig. 7 includes one additional calculation, *7d,* which adds to the configuration of *7a* the simulated intercavitary blood mass described in Fig. 5.

The potential maps in Fig. 8 were calculated with the 30° dipole described in Fig. 4 as a current source. Maps *a, c,* and *d* correspond with maps *a, b,* and *c* of Fig. 6, while map *b* adds lungs to the configuration of *a.*

Before considering the results of the calculations, we would caution the reader to regard the absolute values of the computed potential as meaningless, for the solution of the poisson equation clearly contains an arbitrary constant (*i.e.* the potential is given with respect to an arbitrary ground). Only the difference in the potential of any two surface points has physical significance. Finally, we point out that since it is the voltage of the generator that we consider, the conductivity of the thoracic medium does not enter at all into the calculation for a homogeneous torso, while, when secondary regions are included, only the ratios of the conductivities of the media are relevant.

The following general features may be extracted from the potential maps thus far calculated. First, inhomogeneities can have a drastic effect upon the potential distribution. With the 30° dipole generator, the inclusion of lungs, Figs. 8*b* and 8*d,* altered the potential patterns significantly from those produced in identical homogeneous bodies, Figs. 8*a* and 8*c*. Of particular interest is the fact that the combination of irregular torso cross-section and lungs produces, with this simple dipole generator, a potential distribution displaying secondary maxima and minima characteristics of higher multipole generators (Fig. 8*d*).

The high conductivity blood mass, Fig. 7*d,* also produced a radically different pattern from that of the same generator in the homogeneous torso, Fig. 7*a*. Noteworthy here is the fact that the minimum point, which is above the maximum point in the homogeneous case, moves below the maximum point, nearly to the base cap in fact, when the blood mass is taken into consideration.

When the generator is taken as the 60° dipole, Fig. 6*b*, or as the dipole layer, Fig. 7*b,* the introduction of lungs, Figs. 6*c* and 7*c,* did not produce very different potential patterns.

Even in these last cases, differences become apparent upon detailed examination. For example, the minimum point (with lungs and hemisphere generator) in Fig. 7*c* is shifted up and to the left (to the right on the diagram but to the left with respect to the body) compared to the minimum point without lungs, Fig. 7*b*. Also, the potential gradient is smaller around the maximum point in Fig. 7*c* than in Fig. 7*b*. On the other hand, the 5 per cent change in the value of the potential maximum

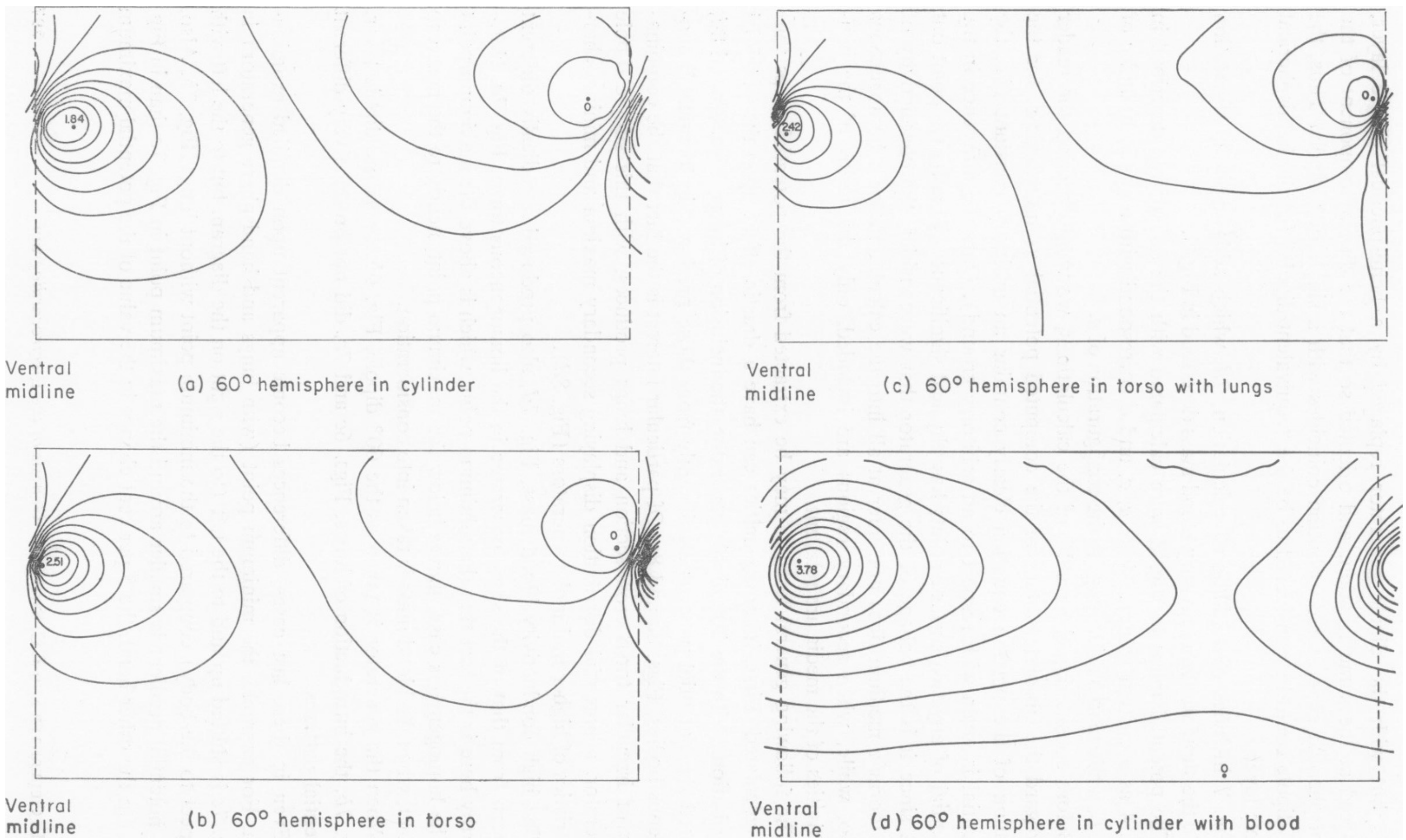

FIGURE 7 Calculated potential maps using as the generator the hemispherical dipole layer described in Fig. 5.

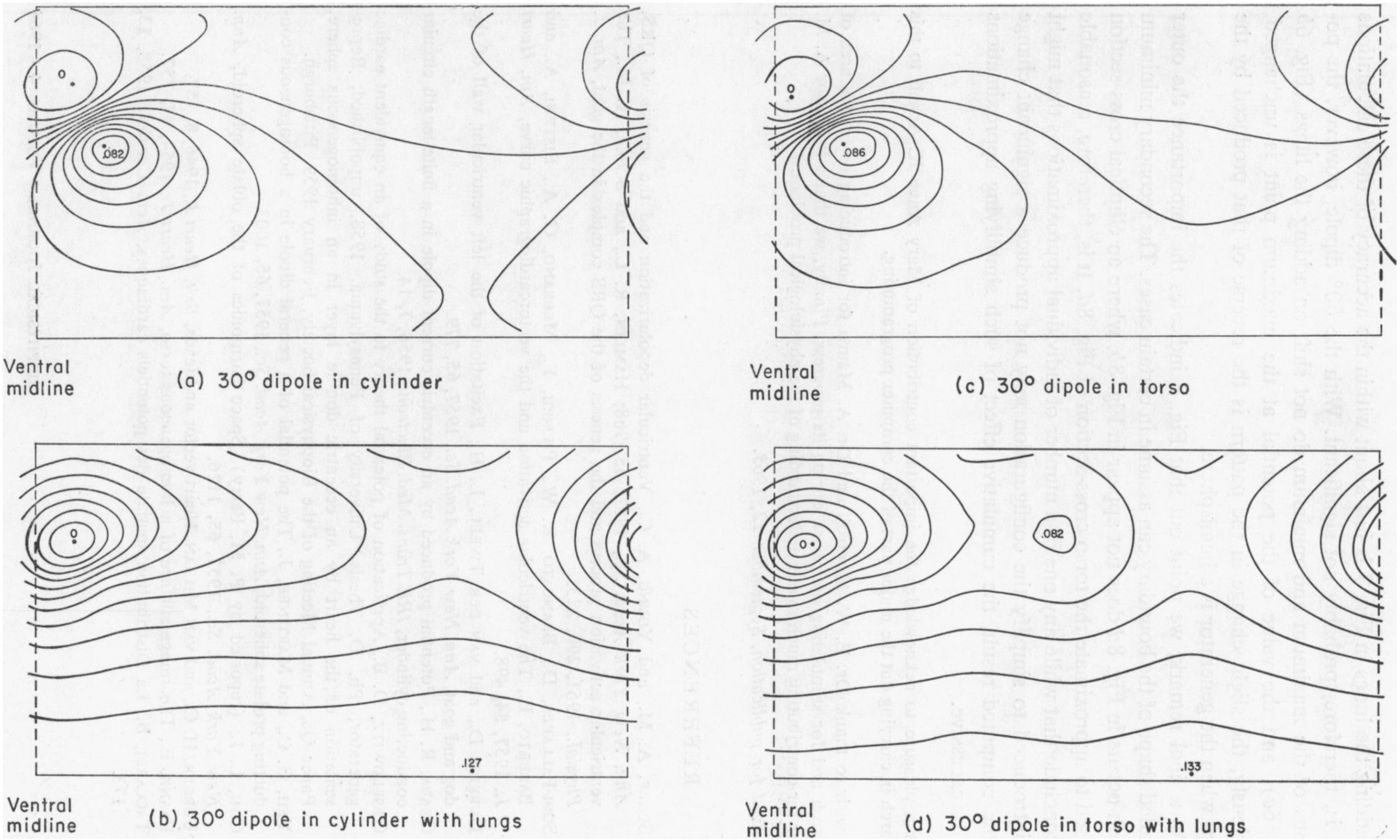

FIGURE 8 Calculated potential maps using the 30° dipole generator described in Fig. 4.

on adding the lungs in Fig. 7 is just about within the accuracy of these calculations and is, therefore, probably not significant. With the 60° dipole, however, the positions of the maximum and minimum do not shift on adding the lungs (Fig. *6b* and *6c*), and the value of the potential at the maximum point is unchanged. Curiously, the slight change in the pattern is the reverse of that produced by the lungs when the generator is a hemisphere.

As a final remark, we point out that Fig. 8 indicates the importance the outer detailed shape of the boundary can assume in certain cases. The secondary minimum which occurs in Fig. *8d* does not appear in Fig. *8b,* where an elliptical cross-section is used to approximate the torso cross-section of Fig. *8d.* It is, therefore, reasonable to conclude that while any one of a number of individual approximations that might be introduced to simplify the configuration may not produce a significant change in the computed result, the cumulative effect of such simplifying approximations can be excessive.

It is a pleasure to acknowledge the important contribution of Mary Anne K. Angell to this research in carrying out the major part of the computer programming.

We wish to thank Dr. E. W. Montroll and Dr. A. Mauro for introducing us to this area of research and for stimulating our work during its progress. Finally, we thank Professor A. M. Scher for contributing much to our understanding of the physiological problem.

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11. After most of the work of this paper had been completed, Professor L. Landweber of the State University of Iowa brought to our attention an unpublished report by J. L. Hess and A. M. O. Smith, Calculation of non-lifting potential flow about arbitrary three-dimensional bodies, Douglas Aircraft Division, Report No. E. S. 40622, 1962. In this report, the same method of computation is applied to the problem of potential flow in fluid dynamics, a problem for which the mathematical description is nearly identical to the electrocardiographic problem for a homogeneous torso. We wish to thank Professor Landweber for pointing out this work.
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